Reprinted from
THE EDUCATIONAL FORUM, May, 1971

Why Prefer the "New Math"?

KENNETH R. CONKLIN

DURING the last ten years there has been a dramatic change in the teaching of mathematics in the elementary and secondary schools. Topics formerly reserved for masters and doctors degree candidates in pure mathematics have now been simplified and introduced into the elementary school curriculum, sometimes even at the first or second grade level. In addition to the introduction of advanced topics at the elementary level, there has been a change in the methods employed in teaching mathematics. Less emphasis is now placed on memorizing formulas or standard problem-solving techniques, while heavy emphasis is placed on intuitive understanding of basic patterns and abstract, deductive reasoning from definitions or axioms to theorems.

New math has largely replaced traditional math in our public schools, although this change has met considerable resistance from parents and teachers who sometimes wonder what was wrong with doing math the way they learned it when they were in school. Why should new math be preferred to traditional math? This question deserves to be answered, for several reasons. Even though we no longer need a rallying cry to spark the now well-entrenched revolution in school mathematics, we do need to reassure parents and teachers that the revolution has been worthwhile. We also have an obligation to keep the revolutionary spirit alive, so that the ideals of the new math do not become lost in the implementation of it. The most brilliant and creative developments in education can quickly become stagnant routines, which are just as dogmatic and destructive of creativity as the old-fashioned system.

Traditionally, mathematics instruction in the public schools has been designed to meet the computational needs of the average citizen. It was thought that abstract mathematical theory would be neither interesting nor useful for the average person, and would be far too difficult for children to understand. Thus, abstract mathematics was confined to the universities, and the only people who encountered it were undergraduate students majoring in pure mathematics or graduate students in mathematics and the physical sciences.

The first defense for traditional math was the claim that students could readily *transfer* what they learned in school to practical situations outside the school. Mathematics in elementary school consisted mainly of practical computation

Although the "new" math is pretty well accepted, very few parents and perhaps not too many teachers can make out a case for preferring it to the traditional arithmetic. This article provides a rationale for such a preference. KENNETH CONKLIN *is Assistant Professor of Educational Studies at Emory University in Atlanta, Georgia. He took his doctoral work at the University of Illinois.*

which would be directly useful in adding up grocery bills, balancing a household budget, figuring out the income tax, or understanding the way insurance premiums are computed. At the secondary level, the character of mathematics became quite different although still computational: students would learn how to solve algebraic equations or how to prove simple measurement theorems in Euclidean geometry. Sometimes these algebra and geometry courses were defended on the grounds that the ordinary citizen would actually need to know how to solve equations or measure areas and spaces. More frequently, though, such courses were defended by claiming that students who mastered the logic of geometry would be more logical about solving the problems of everyday life. It was also claimed that studying something difficult and unpleasant would exercise and toughen the mental faculties, thereby improving the power of the mind to deal with difficult or unpleasant problems in life.

Since both computational skill and mental discipline would transfer to a wide range of situations outside of school, it was argued that every student should study mathematics. *General education* is education which provides knowledge and skill that is necessary or useful for everyone, regardless of his occupation. Since computational skills and mental discipline are both necessary and desirable for the ordinary citizen, and since mathematical training transfers such skills and discipline, it was claimed that mathematics belongs in the general education curriculum for all students.

On the other hand, abstract mathematical theory (set theory, topology, group theory, vector geometry) clearly had no direct practical applications for the ordinary citizen. Topology would not be of much assistance in deciding how to mow the lawn, and group theory or modular arithmetic would only lead to confusion in adding up the grocery bill or figuring out the budget. Abstract mathematics, therefore, clearly did not belong in the general education curriculum. Indeed, the only people who could use abstract mathematics were mathematicians or scientists. *Vocational training*, broadly defined, is any kind of educational program which is undertaken for the primary purpose of producing, maintaining, or improving one's income or one's status in an income-producing occupation. Abstract mathematics, therefore, was thought to belong to the vocational training program of mathematicians and scientists, much as shorthand, speed-typing, and the use of office machines belonged to the vocational training of secretaries.

The major arguments against the new math can be seen quite clearly in this context. Opponents of the new math claim that it represents an unjustifiable shift of topics from a specialized form of vocational training into the general education curriculum. Training in traditional math easily transferred to practical applications for the ordinary person, but instruction in new math only prepares the student for more math or for theoretical science. While it may be true that we need more mathematicians and scientists, it seems unreasonable to demand

that every public school student must become one. Furthermore, students trained in new math devote less time to computational work, and often seem slow or ineffective in solving practical, everyday arithmetic problems. Parents are unable to understand their children's homework assignments, children cannot count apples in the supermarket, and teachers have to go back to school to struggle with some wild, newfangled fad in education.

If new math is to be defended as belonging in the general education curriculum, we must show that it is at least as successful as traditional math in providing the computational skills and the mental discipline needed by the ordinary citizen. In addition, we must show that new math does something which old math could not do, so that it is worth a great deal of effort (not to mention parent-teacher frustration) to substitute new math in place of the traditional approach.

There can be no doubt that a student trained for 12 years in practical computational techniques would do better at practical computations than a student who spends part of that time studying abstract theory. But we may question the extent to which computational techniques are actually used by ordinary people. Certainly everyone should be able to add, subtract, multiply, and divide, and everyone should be able to perform these four operations on positive whole numbers, negative whole numbers, fractions, and decimals. Beyond these four operations on these four types of numbers, there is almost nothing else the ordinary person is called upon to compute. Powers and roots could be safely eliminated, along with all of traditional algebra and all of geometry except the simplest measurement rules for line segments and polygonal areas.

If the new math is deficient in training students to do these simple computations, the deficiency must be remedied. Perhaps the mathematicians and educators who planned the new math programs got "carried away" with the possibilities for doing theory and neglected computational drill. Charges that new math produces children who cannot count apples in a supermarket seem a bit extreme, but it certainly has been a common complaint that children raised on new math just haven't had enough basic computational drill. This defect must be corrected, but the correction will not require great effort or inconvenience. Much time in traditional math programs was wasted learning computational techniques which have no practical applications in ordinary life, e.g., finding square roots, solving quadratic equations, or determining the volume of a truncated pyramid. The gap in computational usefulness between traditional math and new math will therefore not be very hard to overcome.

The claim that traditional math exercised the mind and increased the student's power of logical thinking has come under severe attack by psychologists. The mental discipline or faculty psychology theory has long been unacceptable. Nonetheless, recent theories of stages in cognitive development (e.g.,

Piaget) have suggested that there are certain basic kinds of reasoning that occur only after the child has had an appropriate background of experience together with biological growth. Studying mathematics may indeed facilitate the development of these general reasoning powers, but further research will be needed. It seems safe to say, however, that the new math does at least as well as the old math in promoting the development of general reasoning powers. New math is characterized by abstract thinking, formation of generalizations based on observation and intuition, and deductive reasoning from definitions to proofs—and these are precisely the kinds of mental processes which the latest psychological theories indicate must be practiced by children and early adolescents.

Most teachers have noticed that students are far more enthusiastic about new math than they were about traditional math. New math is interesting, perhaps because it meets the cognitive developmental needs of the students as discussed above. Bright children do better with new math, average children do at least as well, and dull children seem to suffer no more now than formerly. Because almost all students find new math more interesting than traditional math, the job of motivation is made easier and disciplinary problems are reduced.

Thus far we have seen that new math requires only moderate improvement in computational drill in order to be as useful to the ordinary citizen as traditional math, and new math seems to do better at interesting the students and promoting the development of general reasoning powers. New math, improved by computational drill, therefore, is at least as deserving as traditional math to be included in the general education curriculum. But new math offers something very important which traditional math could never begin to approach, and it is this very important something that makes new math worth all the frustrations encountered by parents and teachers.

Not everything in the general education curriculum is intended to be directly applicable to solving the practical problems of coping with the environment. Darwin's law of the survival of the fittest applies to animals whose sole purpose in life is to get along with the environment. Machines can perform many tasks faster and more accurately than man. Yet man is more than an animal or a machine. Man can do more than survive, grow fat for the slaughter, or manipulate the world around him. Man can appreciate his world. Man has an aesthetic sensitivity to the beauty around him enabling him to love and to feel reverence.

Some parts of the general education curriculum are designed to cultivate the distinctively human ability to appreciate. Art, music, literature, and drama have traditionally fulfilled this function. These subjects are not primarily defended on the grounds that everyone needs to know how to draw, play an instrument, write novels, and act. Neither are these subjects defended on the grounds that they help students develop their faculties of mental discipline. Art, music, literature, and drama are appreci-

ation subjects—they help students appreciate the beauty in visual forms, sound, written language, and interpersonal interactions. History also tends to be an appreciation subject if properly taught, giving students a respect for and understanding of civilization and human tradition. The "new" math, "new" science, and "new" grammar are attempts to enhance the appreciation of subjects which are usually defended as being practical or providing mental discipline.

The role of the appreciation dimension of a subject in general education can be better understood if we make a distinction between two ways of using knowledge.[1] Sometimes knowledge is used to reorganize the environment so that a problem is solved. This *applicative* use of knowledge occurs whenever an engineer uses his knowledge of physics to design a bridge, or a student uses his knowledge of the elementary facts of addition to decide whether he can afford to buy all the things he wants. However, knowledge can be used interpretively, to understand a situation or to appreciate a meaning. This *interpretive* use of knowledge occurs when an ordinary citizen uses his knowledge of physics to understand and appreciate the achievements of the space program, even though the citizen could not actually apply his knowledge to design the space ship or compute its fuel requirements.

It is thus possible to have interpretive use of a subject without having applicative use of that subject. Nevertheless, applicative use of a subject requires the previous acquisition of the interpretive use of it. A problem must be understood and interpreted before it can be solved. Vocational training in a subject therefore requires both the interpretive and applicative uses of that subject, while general education in a subject requires only the interpretive use of it. The whole range of general education may be seen as the understanding of a broad spectrum of subjects to be used interpretively, coupled with the development of the ability to cope with generally common environmental problems.

Topics in set theory, topology, group theory, and vector geometry were traditionally limited to university-level vocational training programs for mathematicians and scientists. Knowledge of these topics was meant to be used applicatively, so the topics were studied in depth and with considerable mathematical rigor. In the "new" math these same topics are studied in elementary school as part of the general education program required of all students. Knowledge of these topics is meant to be used interpretively, so the topics are studied with greater flexibility and less emphasis on mathematical rigor.

Interpretive knowledge of mathematics is important for the average citizen in modern America because of the importance of mathematics and science in our civilization. A democracy cannot succeed unless its citizens are sufficiently well informed so that they can intelligently share in the formation of public

[1] Four uses of knowledge, including the applicative and interpretive uses being discussed here, were developed in Harry S. Broudy, B. Othanel Smith, and Joe R. Burnett, *Democracy and Excellence in American Secondary Education* (Chicago: Rand McNally and Co., 1964), Chapters III and IV.

policy. Such a large portion of public policy today is concerned with math and science that being a good citizen requires considerable interpretive use of knowledge in these subjects.

Aside from the requirements of good citizenship, the interpretive knowledge of mathematics is important if life is to be enjoyed to the fullest extent. A person cannot enjoy life fully unless he understands what is going on around him and where he fits into the on-going world. So much of what is taking place today grows out of developments in mathematics and science that awareness and understanding of current events requires the interpretive knowledge of mathematics. Man's greatest achievements today draw heavily upon math and science, so that the proper appreciation of those achievements is possible only through an appreciation of mathematical theory.

Perhaps the most important function of the interpretive knowledge of mathematics is that it makes possible the direct appreciation of mathematics itself. The masterpieces of mathematical theory, like the masterpieces of art, have an inner harmony and an outward magnificence which are awe-inspiring. But the eye must first be trained before it can appreciate beauty. Mathematical beauty is very much like beauty in art, music, or poetry, and those who lack the interpretive knowledge to appreciate it are missing a profoundly meaningful experience.

The "new" math attempts to make the nature of mathematical knowledge highly visible, so that mathematics can be appreciated in the same way as an artistic masterpiece. All the "new" approaches to traditional subjects have in common this effort to lay bare the structure of knowledge and the methods for obtaining knowledge. Curricula in the "new" physics, for example, emphasize the use of scientific method and the relationship between observations and theories. The "new" grammar is concerned with the nature of language as a form of communication and the methods whereby linguistic patterns can be discovered. The "new" history attempts to teach students how historians do research.

Studying the structure of knowledge in a subject is like studying the patterns of color, balance, form, and texture in an artistic masterpiece. Studying the methods whereby knowledge is obtained is like studying the artistic techniques whereby masterpieces are created. Learning about the structure of a product and the method of its production helps one understand and appreciate both the finished product and the process which made it. Interpretive knowledge of product and process is thereby obtained.

When the ordinary person has interpretive knowledge of a product, his enjoyment of that product is enhanced. Interpretive knowledge of a process enables the ordinary person to share in and perhaps control the work of the expert who makes the product. Math, science, language, and the social sciences are so important that general education must include interpretive knowledge for both the humanistic enjoyment of these subjects and enlightened citizen participation in democratic control of the creation and use of subject matter.

In every academic subject, the ex-

perts are divided on a basic philosophical question: Is the subject a study of real things which exist whether or not anyone knows about them, or is the subject something which has been invented by the mind of man and says nothing about outside reality? Are numbers and lines real entities, or have we merely invented them? (No number or line has ever been seen) Do the theorems in mathematics tell truths about the nature of the universe, or are theorems merely true by definition in a fairy-tale system of definitions invented by man?[2]

Philosophers and subject-experts ask the same questions about the status of scientific laws, grammatical patterns, and historical trends. Do scientific laws describe the "personality" of the universe, or are they merely summaries of man's responses and feelings? Is grammatical structure an inborn shaper of thought and perception, or is it merely a set of rules or arbitrary conventions for playing the language game? Are historical trends real and inexorable, or are they man-made myths which summarize nothing real? The complexities of these debates go far beyond the scope of the present discussion, but we may note with pride that new math, new science, new grammar, and new social studies all provide interpretive knowledge which makes the debates much more open to public understanding than formerly.

Although new math has largely replaced traditional math in our public schools, we must continually re-examine the reasons which justify this change. Parents and teachers who understand why new math has been adopted will be more sympathetic toward its purposes and better able to carry out those purposes. Traditional math was defended as part of general education on the grounds that the computational skill and mental discipline it produced were almost automatically transferred to important applications in ordinary life. Topics in abstract mathematical theory were thought to belong solely to the vocational training of mathematicians and scientists.

When compared with traditional math, new math is found to have at least as great a transfer value for mental discipline. Overemphasis on theory has adversely affected the computational skill of new math students, but this defect can be easily remedied by introducing more basic drill. Furthermore, new math provides a very important benefit which was almost totally lacking in traditional math: the interpretive knowledge of mathematical theory. Interpretive knowledge of a subject helps the ordinary person understand and appreciate the role played by that subject in modern life. The well-rounded person needs interpretive knowledge of important subjects in order to enjoy life fully and also to be able to exercise intelligent judgment as a well-informed participant in democratic policy formation.

New math, new science, new grammar, and new social studies all share in a common effort to teach students the structure of knowledge in these subjects and the methods whereby subject-ex-

[2] For a collection of important articles on the existence of mathematical objects and the nature of mathematical truth, see Paul Benacerraf and Hilary Putnam (eds.), *Philosophy of Mathematics* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964).

perts discover this knowledge. Students are thus enabled to appreciate the humanistic and aesthetic dimensions of the subjects they study, and to feel a sense of vicarious participation in the quest for truth. The fundamental philosophical dispute between the realist and conventionalist views on the status of abstract entities is also more open to public scrutiny in the "new" subjects.

For all these reasons, new math should be preferred to traditional math. New math holds great promise as a part of the general education curriculum, but in order to fulfill its promise the new math must be properly taught by teachers who keep in mind its revolutionary spirit. New math is characterized by abstract thinking, formation of generalizations based on observation and intuition, and deductive reasoning from definitions to proofs. All of these things should be done in an open-minded spirit of free inquiry and creative spontaneity. New math must not be allowed to degenerate into the rote memorization of standard rules and procedures which was so typical of traditional math. Teachers can be flexible, creative, and spontaneous only if they are thoroughly trained in abstract mathematics and only if they understand and are inclined to apply the philosophical and psychological bases of the new math.